Magneto Design and Practice Working Paper

The Idealized Magneto

One way to de-mystify the goings-on with magneto and coil design is to start with an idealized model that temporarily skips over some of the nitty-gritty details to get at the main ideas. With a grasp of those main ideas, it is easier to understand the complications that come from using real world materials and geometries. Don't be put off by all the different units of measure. Conversion tables are in Appendix 2.

Basics of the Magnetic Field

Magnetic Flux

Magnetic Flux is best represented in diagrams as a group of continuous lines that flow around a loop and close upon themselves. The word *flux* can just refer in a general way to the "magnetism" that flows in a certain vicinity, but the word is also used to refer to the **measured amount** of flux.

There are two ways to measure flux: Total flux measures all flux linking a particular area, such as a coil. It is represented by the symbol $\boldsymbol{\Phi}$ and is measured in *Maxwells* in the CGS system, as *Webers* in the Metric SI system, or as *Lines* in English Practical units. *Flux density* measures the flux flowing through a tiny local area, divided by that area. It is represented by the symbol $\mathcal{B}$ and is measured as *gauss* (CGS), *teslas* (SI), or *lines/inch*^{2} (English).

Magnetizing Force

Flux can exist in any medium, even in space, but wherever it goes, magnetic flux requires *magnetizing force* to drive it. The local *magnetizing force* at any point in the field is symbolized by $\mathcal{H}$. It is measured in *amp-turns/inch* (English), *amp-turns/meter* (Metric SI) or *oersteds* (CGS). The sum of all the magnetizing force around the whole flux loop is called Magneto Motive Force (MMF) and is symbolized by $\mathcal{F}$. MMF is measured in *ampere turns* in both English Practical and Metric SI units, and in *gilberts* in CGS units.

Reluctance

Reluctance is the measure of how much MMF it takes to drive an amount of flux around a particular loop. It is measured in *amp-turns/weber* in the metric SI system, *gilberts/maxwell* in CGS units, and in *amp-turns/line* in English Practical Units. It is risky to think of it as "ohms law for magnetism," though. Magnetic materials are not linear, and *reluctance* can vary with the amplitude of the flux.

Changing a Magnetic Field

In the following paragraphs we will temporarily ignore resistance in the wires of a coil. Except as noted, we will assume that the flux links (flows through) the coil, and that there are no other active sources of MMF involved.

Rule 1: Total Flux and Applied Voltage

There is a simple but tricky rule that governs the relationship between the magnetic flux linking a coil and the voltage across the terminals of that coil. Unchanging flux generates no coil voltage. **Changes** in the flux will always generate a voltage proportional to the number of turns in the coil times the **rate of change** of the flux. That rule is completely reversible (without resistance in the coil wire). Connecting a voltage across a coil will force the flux in that coil to change so that the rate of change of flux times the number of turns matches the applied voltage.

Rule 2: Total Magnetomotive Force and Coil Current

The MMF required to drive linked flux in a coil is supplied by the ampere-turns of current flowing in the coil. The ampere-turns flowing in the coil must always correspond exactly to the sum of the MMF required to drive the flux around its path.

Interactions

Three examples will show the how flux and MMF interact with coil voltage and current (assuming a zero-resistance coil in a linear magnetic medium):

1. If there is no voltage across the coil then the rate of change of flux must be zero. Flux cannot change, hence neither can the current.
2. If there is a fixed positive or negative dc voltage applied to the coil, the flux will change up or down at a constant rate, and the current will rise or fall to provide the required MMF.
3. Suppose you break open the wire of the coil while flux is flowing. If you try to do that, then the current must stop, which means that the flux must also suddenly stop. But if that happens, the rate of change of the flux is suddenly extremely large, and that forces the voltage across the break in the wire to be extremely large also. Inevitably the gap in the wire or the insulation of the coil flashes over, and the current continues as an electric spark. (Sound familiar?) During the spark the voltage is actually determined by the properties of the arc, and that voltage across the wire break determines the rate of reduction of the flux, hence the rate at which the energy stored in the coil is dissipated in the arc. (Those of you who have looked at this phenomenon on a scope will realize that the voltage and current during the arc are anything but constant. Wild high frequency oscillations are to be expected.)

Thus you can see that the coil voltage is always proportional to the rate of change of flux that links that coil, and the coil current is always proportional to the amount of flux. These two laws of physics are the basic rules. Later we can tie in the effects of coil resistance, iron path nonlinearity, and coupling between separate magnetic fields.

Storing Energy in a Magnetic Field

It takes energy to create a magnetic field linked to a coil, and energy must be released in order to collapse a magnetic field. You can visualize it by imagining a voltage connected across a coil. Current will begin to flow into the coil as the flux increases, and you know that voltage multiplied by current equals power. This input power is delivered to the magnetic field as long as a voltage is applied to the coil. Power delivered over time accumulates as stored energy.

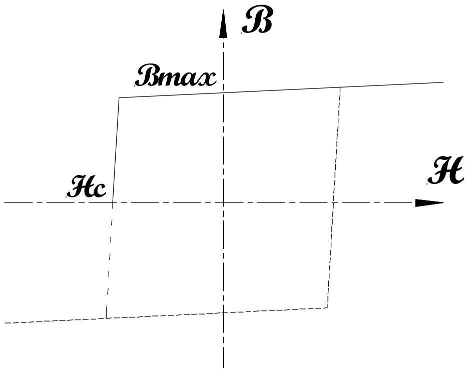
Collapsing a field that links a coil requires delivering energy back out of that field, just the reverse of that very same process. The coil will supply the voltage and the current to an external electrical load of some kind. Energy can be exchanged (in or out) at high voltage for a short time or at low voltage over a longer time.

If you want to make a magneto, you'll want to take note of this: moving an external magnetic field (or moving the coil) to link an open-circuited coil does not store energy in the coil itself. The changing flux linked to the coil definitely generates a coil voltage, but because no current flows the coil provides none of the MMF, and no energy is exchanged. You might almost say that the coil does not really exist at all if it is an open circuit.

Moving a magnetic field linking a short-circuited coil is a different story. Since the coil is shorted, linked flux within the coil can't change, so the coil is forced to generate an MMF to compensate for any change in the external field. Thus current is generated in the coil. It can be shown that mechanical force is required to move the magnetic field (or the coil) and store energy in the field linked to the coil as described here.

Permanent Magnets

If you read about permanent magnets, you will encounter terms like "square loop," saturation flux density, and coercive force. We will confine the discussion to really "hard" (really square) magnetic materials such as neodymium "super magnets."



Material within the volume of a super magnet will have a built-in MMF field measured in units of ampere-turns per inch, or per meter, or *oersteds*, depending on the system of measurements being used. This is called the (just plain) *coercive force*, or $\mathcal{H}_c$ of the magnet. See the diagram on the right. Multiply this by the length of the magnet, and you will know the ampere-turns of MMF that magnet will contribute to a magnetic loop.

That MMF will cause flux to flow around an external loop, until the back-MMF of the external flux path equals the magnet's MMF. The *flux density*, $\mathcal{B}$, within the magnet will be that flux divided by the cross sectional area of the magnet. The maximum total self-generated flux available from the magnet will be limited by the maximum flux density of the material, or $\mathcal{B}_{max}$. Both $\mathcal{H}$-limited or $\mathcal{B}$-limited equilibrium conditions fall along the left-most and top-most lines on the $\mathcal{B}$-$\mathcal{H}$ loop shown in the figure above.

If a supermagnet is placed in a very large external magnetic field, the residual magnetism in the device can change. The field intensity required to change the state of the magnet is called the *intrinsic coercive force*, only slightly larger than the (just plain) *coercive force*. Permanent magnets are "charged" by applying the charging field along the desired axis of the magnet.

The maximum flux density in a permanent magnet is a little less than the saturation flux density in transformer iron. In N40 Neodymium material the residual flux density is about 1.27 *maxwell/m*^{2}, compared to 1.55 *maxwell/m*^{2} for typical transformer iron.

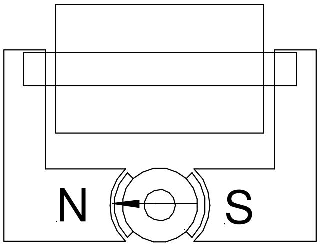
The *coercive force* for N40 material is an eyebrow-raising 875,000 *amp-turn/m*, compared to the saturation threshold value for transformer iron of about 1575 *amp-turn/m*. Remember, the coercive force represents the MMF that the magnet contributes to the whole loop of the magnetic field. The actual MMF in a magnetic circuit includes the magnet itself plus the MMF contributions of all other elements of the flux path.

The Ideal Magneto

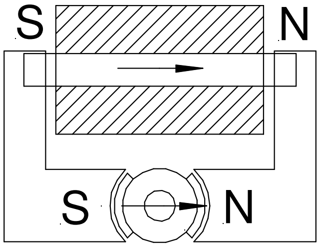
The basic ideas of how magnetos work are understandable enough, but they are not necessarily intuitive. A lot of what has been written describing magnetos during the past century has been vague and imprecise, even occasionally false and misleading. Here is my perception of how things really work.

Initial Magnetic Field

Start with the rotor aligned as shown at right. The coil is open, so the coil core is just a piece of iron that provides a path for the rotor flux to complete its loop. Assume here the coil core is not saturated and is a much better path than air, so all the flux is in the iron.



Using my salient pole magneto as a numerical example, the rotor magnet can supply about 90 micro-webers of flux flowing around the loop.



Rotor Field Reversal

Now close the points to short the coil and flip the rotor over 180 degrees. Here's what you have:

1. Rotor flux is now flowing left to right (out the north pole, in the south pole).
2. Because the coil is shorted, its flux is "frozen" as it was before—the same amount as the rotor flux—flowing left to right through the coil.
3. There is no iron path for the combined coil and rotor fluxes to get from the north poles back to the south poles. Instead, it must fringe out and return around the rotor and the coil through air.

Current in the Shorted Coil

Remember that the MMF required to drive the flux is determined by the reluctance summed around the flux path. Since that path is now largely through air, you can guess that the required MMF is high. Calculating the actual path reluctance is a job for NASA computers, but we can make a guess that will at least nail down the order of magnitude. In my salient pole magneto my guess at the air gap (0.02 m long, 0.05 m x 0.05 m cross section) gives a final MMF of 1150 amp turns. That MMF requires 4.6 amps in a 250 turn coil. See the appendix for more details on this calculation.

Ready to Fire

The amount of energy stored in the coil is relatively easy to calculate from the information we already have. It is simply 1/2 times the coil flux times the MMF. In this case, that turns out to be about 52 millijoules, which is between 2 and 5 times more than we would really need to make a reliable spark in a model engine.

With all that current flowing in the coil, we now have only to open the points to release it into a spark discharge, as previously described. In a low-tension ignition system with a set of ignitor points operating inside the cylinder, that would be it. In a high-tension system, of course, the coil has a secondary winding to step the voltage up further to fire a spark plug.

The bad news is we don't really get to operate an ideal magneto. We have to deal with losses along the way that eat considerably into our potential cache of stored energy.

The Real Magneto

The basic ideas of how magnetos work, including the electromagnetic laws of physics just discussed, don't change when you have to deal with real world materials and techniques. They do become hard to apply and interpret sometimes. Some of the things that cause performance loss (and make it harder to understand what is going on) are described next:

Lost Energy

Most everything that happens in the real world dissipates energy. Just about everything that happens in a magneto to cause variations from the ideal involve loss of energy. The losses that seem most important in the magneto are listed here:

Coil Core Saturation

In the description of the *Initial Magnetic Field* of the ideal magneto, we assumed that the iron core of the coil would be large enough to allow all the rotor magnet flux to flow though it, limited by the saturation flux of the magnet itself. If the coil core iron saturates, obviously there will be less flux in the coil when the rotor flux reversal takes place.

In my Salient Pole magneto example, the available rotor flux was about 90 micro-webers. I calculate the saturation flux of my coil core to be about 62 micro-webers, so only about 2/3 of the rotor magnet flux is flowing through the coil. The stored energy after rotor flux reversal is 30 *mjoules*, so this limitation cuts available spark energy nearly in half. See the appendix for an example of calculations allowing for this decreased flux. The moral of the story is that using a big rotor magnet with a small coil core does increase the stored energy, but is not a very efficient way to do so.

Coil Resistance Losses

We already know that a shorted zero-resistance coil linked to a magnetic field will maintain the field and the current and the stored energy in the field indefinitely. Since a real coil has resistance, however, there is a voltage drop across that resistance. Voltage drop is distributed along the wire, but still creates a rate of change of flux, in this case negative. Therefore flux decays and the magnetic field energy is dissipated in the coil resistance over time.

As we saw with the ideal magneto, energy is stored in the coil as flux reverses with the points closed. If that flux reversal stretches out over time (and it always does) the energy will accumulate as the flux changes, but at the same time it is also decaying. It becomes a race. The slower the flux reversal, the more time there is for energy to dissipate. At a slow enough speed, this coil energy loss becomes the dominant factor in magneto performance.

I won't do the proof in this paper, but it can be shown that reducing the rate of energy loss requires increasing the total amount of copper in the coil primary. It really does not matter if you use larger wire for the same number of turns or if you use more turns of the same sized wire. I've measured dramatic improvements in my salient pole magneto at 300 rpm by increasing the percentage of the coil's volume devoted to the primary winding.

Ignition Point Phasing

I call setting the timing of the points relative to the rotor *ignition point phasing* to avoid confusion with engine timing. The maximum spark energy will occur if the points open just at the peak of the current flow in the coil primary. In the ideal magneto, there is no decay, so the peak current will occur after the rotor flux has completely reversed. In the real magneto, the faster the current decays the earlier the peak current will occur. This is a fairly broad peak, and timing is not super-critical, but large misadjustment of ignition point phasing will greatly reduce magneto output. The greatest impact occurs at the lower operating speed.

I usually set the point phasing by running the magneto at the lowest design speed and observing the coil current with an oscilloscope using a very low resistance current probe shunted across the points. Phasing can be set also be set by trial and error, although it is nearly impossible to make judgments about small changes in the strength of a spark just by observing it. I start by setting points to open about 120 degrees after initial pole alignment. Then I bracket the correct setting by looking for a substantial fall-off from mis-setting the phasing in both directions. The final setting is half way between.

Ignition Point Arcing

Ignition points always arc—at least a little bit— when they open. They have to, because the initial gap as they start to open in so infinitesimally small, there is no way to avoid it. The trick is to avoid dissipating a lot of energy in the points and to develop enough voltage to fire the spark plug. Once the plug fires it draws the bulk of the coil energy away from the points. This is a very complex little symphony of simultaneous goings-on, and I've given up hope of completely understanding it. A few generalizations are all I can offer here:

1. Sparks have a very peculiar little quirk: the more current in the arc, the lower the voltage becomes. I believe this is because the spark gets hotter so the plasma of the arc has more free ions and electrons to conduct the current.
2. The capacitor plays a key role. It diverts at least part of the current from the points, thus (I believe) reducing the conductivity of the arc. In addition, this diverted current stores energy in the capacitor which is not lost, but eventually dumps into the spark plug.
3. A large turns ratio in the coil helps a great deal. I presume it provides more voltage to initiate the spark at the plug. And when the plug does fire, the voltage at the points is lower because of the turns ratio.
4. High energy in the magneto helps also. It apparently makes it more likely for the plug to develop a good low-resistance plasma path to reduce voltage at the points.
5. Capacitor size variations affect the results, but in my experiments, the exact value is not very critical.

Iron Losses

Iron losses are perhaps less significant than some of the other effects we have been discussing, but they exist:

1. Eddy current loss occurs because iron itself is a conductor. In effect, the iron forms a shorted turn around itself. The effective cure is to use laminated iron to carry changing flux. Laminations break up the shorted turns in the iron. Just make sure you don't do things like weld laminations together at more than one point. Note that for places like the rotor poles where the flux is unidirectional, solid iron is just fine; laminations offer no advantage.
2. Hysteresis Loss is a property of the iron material itself. It's a little like friction. Whenever you change flux it always takes a little energy to turn those molecules around. The best you can do here is use transformer iron, which is engineered to minimize hysteresis. Some audio transformer cores actually have very low hysteresis, and would be preferred for that reason, except that these also have a lower saturation flux density, so have to be sized larger for a given amount of flux. I have not experimented with ferrite or other ceramic material, so can offer no guidance there.

Other Parasitic Losses

Watch out for magneto structure that connects any kind of metal completely around the iron path that contains changing flux. A magneto frame with a "shorted turn" around a side pole, or even part of a side pole, can completely block that leg of the magnetic path if the resistance around that leg is low enough. Fasteners through the center of a magnetic pole piece don't seem to have much effect, but make sure the remaining iron cross section is large enough, and make sure you don't have a "shorted turn" around half of the pole piece because the fastener completes a turn through a "U" channel, for example.

Corona losses from high voltage near the surface of the coil to a grounded frame part might go unnoticed, but could eventually burn a channel in the insulation and cause a failure.

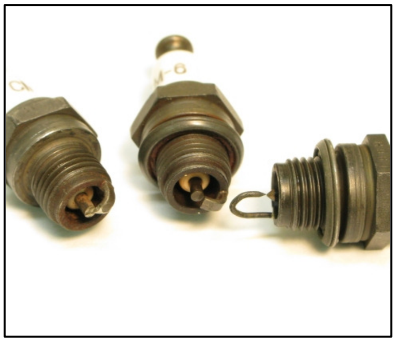
Capacitor Selection

I'm afraid I don't have much science to offer with respect to selection of capacitor sizes. The typical value I've seen used is 0.10 μf. In my experiments cutting this value in half or doubling it has produced very little change in performance. My guess is that for ignition systems with 2.0 to 4.0 amps peak current, this value is appropriate. If you use a primary winding with more turns, you will get smaller peak current (for the same MMF in the coil) and in that case you may want to try a 0.05 μf instead. Automobile capacitors I've measured seem to be in the range of 0.20 to 0.25 μf.

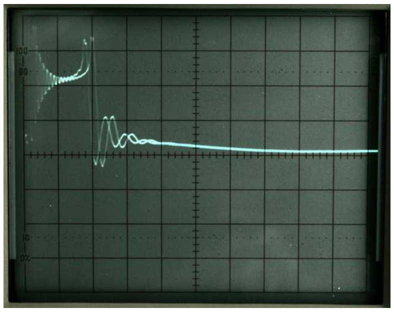
Ceramic capacitors are widely used, although I was taught that they are not very reliable in applications where they have high peak currents flowing in them, as ignition capacitors do. I use Mylar capacitors in my magnetos. They see high peak voltages, so a voltage rating of 100 vdc would be the absolute minimum for long term reliability.

Spark Plugs

I've written another paper on spark plug testing, and will only summarize briefly what I've found with the limited testing I've done so far:



1. Low energy sparks from the ignition source result in random misfires. Stable, repeatable sparks at atmospheric pressure require from 3.5 to 4.5 *mjoules* of energy delivered to the spark plug.
2. Compression pressure raises this minimum energy dramatically. At 100 psi pressure in air, the required energy varies with electrode shape, but ranges between 15 and 25 *mjoules*.
3. I don't regard peak spark voltage to be a good indicator of magneto performance. High peak voltage will fire the plug quickly, but it takes energy to make a stable spark and to ignite a fuel mixture.
4. Having said that, I cannot measure energy directly. Instead I calculate the known energy stored in the ignition coil prior to the opening of the points. I also visually inspect the voltage waveform at the coil primary to look for a stable, repeatable spark. The picture at right (double exposure with two consecutive cycles) shows the characteristic burst of oscillation, followed by a dip in the voltage indicative of a stable spark, followed by ringing as the spark breaks up.



5. I do measure secondary current through a shunt resistor at the base of the spark plug. This current shows longer duration but lower current for secondaries having more turns, as you would expect. The current is a sawtooth form that typically starts from 10 to 20 milliamps and decays linearly to zero.
6. Double-sharpened spark plugs, such as used in some model airplane engines, require the least firing energy
7. A good compromise—and the one I use—is a stock spark plug with a round, flat center electrode and a truncated ground electrode filed to a sharp, straight edge just above the middle of the center electrode. See the center plug in the picture at right.

Transformer Coupling Rotor Current into an Established Spark

We have not discussed it in this paper, but I have seen a second method of coupling energy from the rotor magnet into the spark. It first became obvious in a magneto that John Vietti sent me for testing. This was a large magneto with an excess of flux and a large number of turns on the coil secondary. The coil was supplied by John Rex.

When running the test, I found an exceptionally long spark, persisting for over 2 milliseconds. That was almost an order of magnitude longer than the other magnetos I was testing at the time. There were other indications from the primary voltage waveform that something unusual was happening.

I'll skip over the details here and jump straight to the bottom line: After the stored energy in the coil had initiated the spark in the plug, the rotor flux reversal was still taking place, and the induced *secondary* voltage resulting from the still-changing flux was sufficient to sustain the spark directly. The coil was simply acting as a transformer, coupling the electromechanical energy of the spinning rotor directly into the spark. It's an interesting phenomenon, to say the least, worthy of further thought some day.

What Design Practices Seem to Work Best?

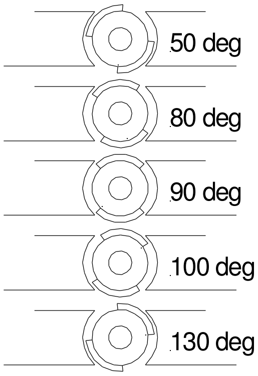
The following recommendations are opinions stated here for the purpose of generating critique and discussion. I expect to learn more than anyone else from those discussions.

Basic Magnetics

This section is just a review of the techniques I have used or seen used by John or others.

Magnets and Rotor Design

A strong magnet is an important asset in designing a miniature magneto. Yet the strength of the magnet can only be used to generate a spark if the iron path, including the coil core is able to carry the flux from that magnet. There is no particular harm in using outsized magnets, but the combination is inefficient.



There is a lot to be gained by shaping the rotor field so that it reverses as quickly as possible as it rotates. This is particularly beneficial if the magneto is to work at slow speeds. The diagram at right shows the design of my salient pole rotor. As you can see, most of the reversal of the flux into the pole pieces takes place between 80 degrees and 100 degrees of rotation. Another form of speed-up is to use a four-pole, six-pole, or even an eight pole rotor. You would do this anyhow for a multi-cylinder engine.

There are other possibilities as well. Some have used what has been called "inductor magnetos," or as I call them, "variable reluctance magnetos." In these the coil(s) and the magnet(s) are both stationary, and a cogged wheel serves as a rotor. Its function is to switch flux back and forth through the coil or coils. These have some real possibilities, but I have not studied them in any detail.

The amount of flux available from the rotor is proportional to the cross sectional area of the magnet that supplies the flux, subject to any leakage flux that spills between rotor poles in one way or another. The diametrically-charged sleeve magnet used by several—John Vietti, Hans Mueller, and myself—has quite a bit of cross sectional area because both sides of the cylindrical magnet carry flux in parallel (perpendicular to the axis). It would take a 0.375 inch rod magnet to equal the cross section of the 0.450 diameter x 0.550 length sleeve. You can calculate the actual amount of flux in the magnet if you want, but it is sufficient to just know the cross sectional area of the magnet in order to compare it against the size the coil core. Read on…

Coil Core Size and Shape

You can compare the cross sectional area of the coil core with that of the rotor magnet. If they are the same, the coil core will not saturate. It is perfectly acceptable if the magneto design does not permit the coil and rotor magnet to be matched in size, but of course the smaller of the two will determine the maximum available output energy of the magneto.

The ratio of coil length to diameter will affect the leakage flux linking the coil, but for an unsaturated iron-core coil, the difference between short and long is not significant unless carried to extremes. The size and shape of the magneto and the space needed to wind the coil will generally determine the coil length.

Number of Turns in Coil Windings

In the mathematics of the magneto analysis, the important things that happen to the primary winding always comes out in term of ampere-turns. It doesn't make a bit of theoretical difference if you put a lot of current through a few turns or a little current through a lot of turns. However, in the practical world the number of turns is does matter, for nuts-and-bolts reasons. Some of the things that make a difference are:

1. The current through the points is inversely proportional to the number of turns in the primary winding. My experience with miniature magneto points is that they seem happiest with somewhere between 2 and 4 amps peak current. Peak coil current varies with speed and with point phasing, and is hard to predict in any particular magneto design. Sometime the easiest approach is to wind a primary-only test coil with a trial number of turns and test it in the magneto itself.
2. A related factor is the resistance of the points and the wire that connects them across the coil. That resistance will always cause some energy loss before the points open. That loss is much more important with a small number of turns and a large primary current.
3. The voltage across the points will be larger if the number of turns is larger. Also, with more primary turns for a given secondary, the turns ratio is decreased. Both factors combine to make sparking at the points more likely.
4. The size and flexibility of the wire are a consideration when it comes to actually laying the windings onto a small coil.
5. In any case, the volume of copper in the primary should be around 50%; it should be as great as 65% if the magneto is to perform well at low speeds.

I don't believe the choice of wire size or number of turns is all that critical so long at the total volume of copper on the coil is about right. For the few coils I have made for my salient pole magneto, I find that about 230 turns of #23 wire gives a good result.

Turns Ratio

I don't have a good theoretical argument in favor of a large turns ratio, but John Vietti and I have both concluded that higher turns ratios give more reliable sparks. In measuring spark plug current I find, as you would expect with any transformer, that large turns ratio coils have lower spark plug current, but the energy stored in the magneto is the same, so the sparks last longer.

From a very small sample of trials, I conclude that a turns ratio less than 50:1 is clearly a disadvantage for a coil, improving with a larger ratio at least up to a 100:1. My latest coil designs have about 75:1, which is a practical compromise. Adding secondary turns beyond 15,000 to 20,000 becomes increasingly difficult, considering that wire sizes smaller than #48 or #50 are fragile to work with.

Revision Notes 6/21/2012:

I have removed prior references to superconductor coils because the analogy was not as useful as I had thought in explaining the basic operation, and was a distraction. I also revised the description of permanent magnets to correct a misconception that I had when I wrote the original paper. These led to widespread editing to improve clarity, but no substantive changes. Finally, I corrected a decimal error in the calculation of the maximum rotor MMF and calculated the reduced spark energy to take coil core saturation into account on page 10.

Appendix 1: Calculations for this paper

Idealized Magneto Example

Rotor Flux

Core description: cylindrical, 0.450 OD, 0.250 ID, 0.550 length, in inches.
Flux path cross section: (OD - ID) x Length = 0.110 $\text{in}^2 = 71 \times 10^{-6}\ \text{m}^2$.
Flux path length (estimated from geometry): 75% OD = 0.35 in = 0.009 m.
Rotor Material Neodymium N40:
$\mathcal{B}_R$ = 12.7 kGauss = 1.27 Webers/m^2; HC = 11 kOersted = 875 kamp-turn/m.
Saturation Flux = $\mathcal{B}_R$ x cross section = $1.27 \times 71 \times 10^{-6}$ = **90 μweber**.
Maximum MMF = $\mathcal{H}_C$ x Flux Path Length = $875 \times 10^3 \times 0.009$ = **7880 amp-turn $_{\text{max}}$.**

Maximum MMF after Coil Reversal

Assumed air gap flux path 2 inch square area 3/4 inch length.
Flux path cross section: $2^2 = 4\ \text{in}^2 = 0.0025\ \text{m}^2$.
Flux path length: 3/4 in = 0.02 m.
Total flux = 180 μweber (twice the maximum magnet flux)
$\mathcal{B}_{\text{air gap}} = 180 \times 10^{-6} / 0.0025 = 0.072\ \text{webers/m}^2$
$\mu_0 = 4\pi \times 10^{-7}$.
$\mathcal{H}_{\text{air gap}} = \mathcal{B}_{\text{air gap}} / \mu_0 = 0.072 / 4\pi \times 10^{-7}$ = 57300 amp-turn/m.
$\text{MMF}_{\text{air gap}} = \mathcal{H}_{\text{air gap}} \times \text{length}_{\text{air gap}}$ = 1150 amp-turns.

Stored Energy

W = 1/2(Flux x MMF) = $1/2(90 \times 10^{-6} \times 1150)$ = 52 mjoules.

Real Magneto Example

Coil Core Flux at Saturation

Core Description: square 0.25 in, length 1.5 in
Flux path cross section: $0.25^2 = .0625\ \text{in}^2 = 40 \times 10^{-6}\ \text{m}^2$.
Flux path length 1.5 in = 0.0381 m.
Core material Si transformer iron:
$\mathcal{B}_{\text{sat}} = 1 \times 10^5\ \text{lines/in}^2 = 1.55\ \text{Webers/m}^2$;
$\mathcal{H}_{\text{sat}}$ = 40 amp-turn/in = 1.6 kamp-turn/m.
Coil $\text{Flux}_{\text{sat}} = 1.55 \times 40 \times 10^{-6}$ = **62 μweber**.
Coil $\text{MMF}_{\text{sat}} = 1600 \times 0.0381$ = **61 amp-turn $_{\text{sat}}$.**
Air gap flux = coil flux + magnet flux = (90 + 62) = **152 μweber**
$\mathcal{B}_{\text{air gap}} = 152 \times 10^{-6} / 0.0025 = 0.0608\ \text{webers/m}^2$.
$\mathcal{H}_{\text{air gap}} = \mathcal{B}_{\text{air gap}} / \mu_0 = 0.0608 / 4\pi \times 1^{-7}$ = 48400 amp-turn/m.
$\text{MMF}_{\text{air gap}} = \mathcal{H}_{\text{air gap}} \times \text{length}_{\text{air gap}}$ = 976 amp-turns.
Total coil current = $(\text{MMF}_{\text{air gap}} + \text{MMF}_{\text{coil}})$ / turns = (976 + 61) / 250 = **4.15 amp**.
W = 1/2(Flux x MMF) = $1/2(62 \times 10^{-6} \times 976)$ = **30 mjoules**.

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Appendix 2: Electromagnetic Units

Parameter		English		Metric			
		Practical		CGS emu		SI	
	symbol	Unit	Value	Unit	Value	Unit	Value
Voltage	V	volt	1	abvolt	10^8	volt	1
Current	I	ampere	1	abamp	10^{-1}	ampere	1
Resistance	R	ohm	1	abohm	10^9	ohm	1
Magnetic Flux	ϕ	line	1	maxwell	1	weber	10^{-8}
Magnetomotive Force	$\mathcal{F}$	amp-turn	1	gilbert	1.2566	amp-turn	1
Reluctance	$\mathcal{R}$	amp-turn/line	1	gilbert/maxwell	1.2566	amp-turn/weber	10^8
Flux Density	$\mathcal{B}$	line/in^2	1	gauss = maxwell/cm^2	0.1550	tesla = weber/m^2	1.55×10^{-5}
Permeability space	μ_0	line/amp-turn-in	1	gauss/oersted	1	weber/amp-turn-m	$4\pi \times 10^{-7}$
Magnetizing Force	$\mathcal{H}$	amp-turn/in	1	oersted = gilbert/cm	0.4947	amp-turn/m	39.37

Equations for cgs emu units:

$\mathcal{R} = \frac{l}{\mu A}$, reluctance, where μ may be a function of flux density.

$\mathcal{F} = 4\pi NI,$ mmf in gilberts, for current in abamperes.

$\phi = \frac{\mathcal{F}}{\mathcal{R}} = \frac{4\pi NI}{l/\mu A} = \frac{4\pi NIA}{l}\mu$, flux in maxwells, current in abamperes.

$\mathcal{B} = \frac{\phi}{A} = \frac{4\pi NI}{l}\mu$, flux density in gauss and current in abamperes.

$\mathcal{H} = \frac{4\pi NI}{l}$, magnetic field intensity in oersteds and current in abamperes.

$\mathcal{B} = \mu\mathcal{H}$

$L = \frac{N\phi}{I} \times 10^{-9}$, inductance in henries, flux in maxwells, current in abamperes.

$W = \frac{1}{2}\phi\,\mathcal{F}$, energy in ergs, flux in maxwells, mmf in gilberts.

Equations for Metric SI units:

$\mathcal{R} = \frac{l}{\mu A}$, reluctance, where μ may be a function of flux density.

$\mathcal{F} = NI,$ mmf in amp-turns, for current in amperes.

$\phi = \frac{\mathcal{F}}{\mathcal{R}} = \frac{NI}{l/\mu A} = \frac{NIA}{l}\mu$, flux in webers, current in amperes.

$\mathcal{B} = \frac{\phi}{A} = \frac{NI}{l}\mu$, flux density in webers/m^2, current in amperes.

$\mathcal{H} = \frac{NI}{l}$, magnetic field intensity in ampere-turns/m, current in amperes.

$\mathcal{B} = \mu\mathcal{H}$

$L = \frac{N\phi}{I}$, inductance in henries, flux in webers, current in amperes.

$W = \frac{1}{2}\phi\,\mathcal{F}$, energy in joules, flux in webers, mmf in amp-turns.